

# Polymer Materials

## Lecture 5: Molecular weight of polymer

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# Molecular weight of polymers

Short polymers : Molecular weight: 100 – 1000 g/mol

(1000 g/mol: waxes, soft resins)

Long polymers: 10,000 to several million g/mol – solids

- Long chain → increased **bonding** between molecules.

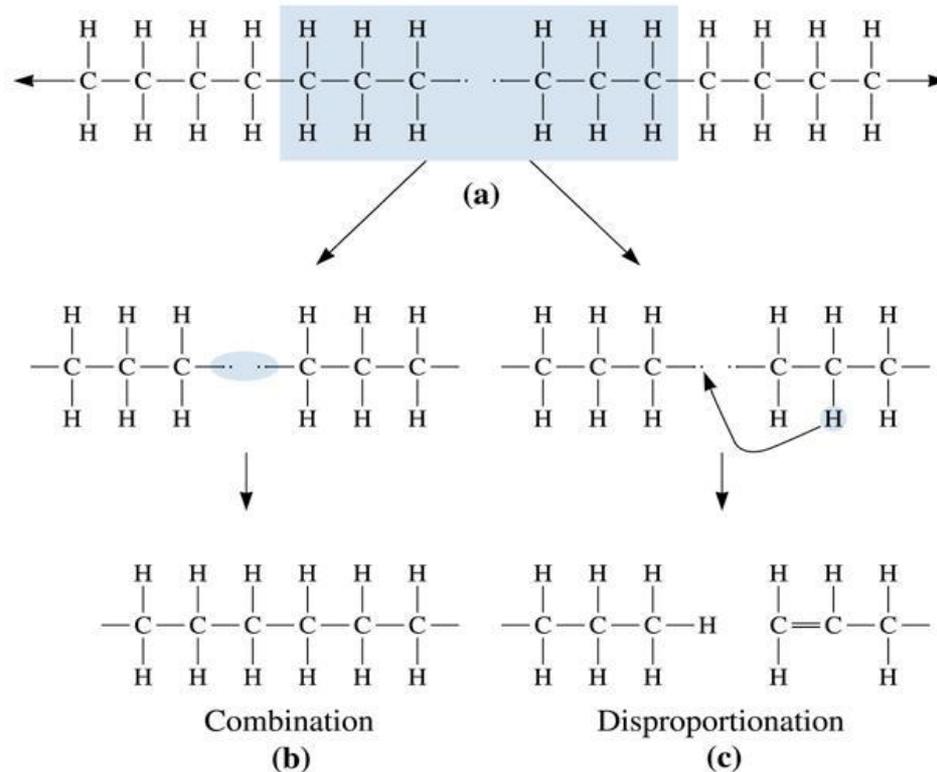
(Van der Waals or hydrogen bond) → **higher** mechanical properties

- Melting point increases with molecular weight

i.e. increased intermolecular forces

# Why polymer has different molecular weight?

- During termination of polymerization, it can be combination or disproportionation polymers
- Different size → **different** molecular weight



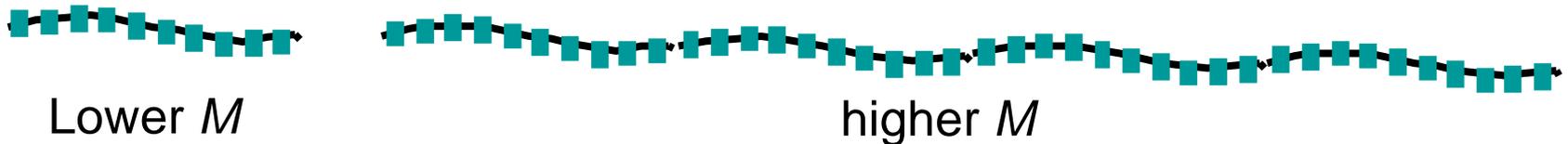
# Number and weight average molecular Weights

Definition:

(1) Number average molecular weight ( $M_n$ ) = based on **number fraction**

(2) Weight average molecular weight ( $M_w$ ) = based on **weight fraction**

The length of the linear polymers can be varied largely  
Different length  $\rightarrow$  different molecular weight  $M_i$ .



## $M_n$ : Number-Average Mol. Wgt.

- The number-average molecular weight (molar mass) of a polymer containing  $N_i$  molecules of mass  $M_i$  is the arithmetic mean of the molar mass distribution:

$$M_n = \frac{\sum N_i M_i}{\sum N_i}$$

- $M_n$  determines the polymer's **colligative properties** (boiling point, melting point changes due to solute **concentration**)
- $M_n$  may be determined directly by **end-group analysis, osmometry, ebullioscopy** (bp elevation), and **cryoscopy** (fp depression).

## $M_w$ : Weight-Average Mol. Wgt.

- The weight-average molecular weight (molar mass) is the sum of the products of the molar mass of each fraction multiplied by its weight fraction ( $w_i$ ).

In terms of  $w_i$  or numbers of molecules,  $M_w$  is

$$M_w = \sum w_i M_i$$

- $M_w$  accounts for the **distribution** of molar mass in the polymer.
- $M_w$  may be determined directly by **light scattering**.

# Molecular Weight Distribution

- The molecular weight distribution, or **polydispersity index**, is the ratio of the weight-average molecular weight to the number-average molecular weight:

$$\text{PDI} = \frac{M_w}{M_n}$$

- The polydispersity index of a monodisperse polymer is 1.00.
- The polydispersity index increases as the polymer distribution broadens.

# Number and weight average molecular Weights

The number average is always **smaller** than weight average.

$$\bar{M}_n = \frac{\text{total wt of polymer}}{\text{total \# of molecules}}$$

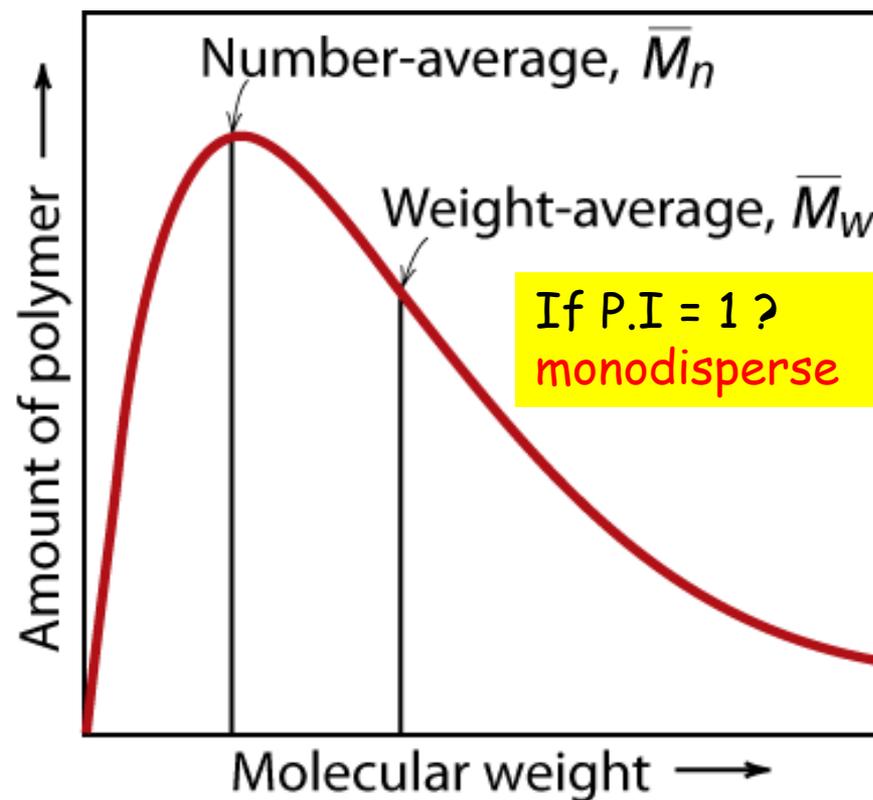
$$\bar{M}_n = \sum n_i M_i$$

$$\bar{M}_w = \sum w_i M_i$$

$M_i$  is the mass of chain

$n_i$  is the number fraction

$w_i$  is the weight fraction



Adapted from Fig. 14.4, Callister 7e.

# Example

You have a polymer sample that contains the following molecules:

$M, \text{Da}^*$	$N$
1,000,000	2
700,000	5
400,000	10
100,000	4
50,000	2
Total:	23

\*Da = dalton, g/mol

What are  $M_n$ ,  $M_w$ , and the polydispersity index?

# Solution

$$M_n = \frac{\sum N_i M_i}{\sum N_i}$$

$$M_n = \frac{10,000,000 \text{ Da}}{23}$$

$$M_n = 435,000 \text{ Da}$$

$$M_w = \sum w_i M_i$$

$$w_i = \frac{N_i M_i}{\sum N_i M_i}$$

$$M_w = 609,500 \text{ Da}$$

$$PDI = \frac{M_w}{M_n}$$

$$PDI = \frac{609,500 \text{ Da}}{435,000 \text{ Da}}$$

$$PDI = 1.40$$

$M, \text{ Da}$	$N$	$N \cdot M, \text{ Da}$	$w$	$w \cdot M, \text{ Da}$
1,000,000	2	2,000,000	0.20	200,000
700,000	5	3,500,000	0.35	245,000
400,000	10	4,000,000	0.40	160,000
100,000	4	400,000	0.04	4,000
50,000	2	100,000	0.01	500
<b>Totals:</b>	<b>23</b>	<b>10,000,000</b>	<b>1.00</b>	<b>609,500</b>

$$w = \frac{2,000,000}{10,000,000} = 0.2$$

# Degree of polymerization (n)

How many units of monomer in a polymer?

$$\overline{M}_n = \frac{\text{total wt of polymer}}{\text{total \# of molecules}}$$

$n_n$  = Number average degree of polymerization

$$n_n = \frac{\overline{M}_n}{\overline{m}} \quad \overline{m} = \text{Monomer's molecular weight}$$

## Example 2:

- Polyvinyl chloride (PVC) sample containing main chains with different molecular weight
  - 4000 chains of molecular weights of 0 – 5000 g/mol
  - 8000 chains of molecular weights of 5000 – 10,000 g/mol
  - 7000 chains of molecular weights of 10,000 – 15,000 g/mol
  - 2000 chains of molecular weights of 15,000 – 20,000 g/mol
- Determine:
  - (1) number average molecular weights
  - (2) weight average molecular weights
  - (3) degree of polymerization

# Answer

- (1) Calculated the number average molecular weight

Number of Chain	Average weight ( $M_i$ )	Number fraction ( $x_i$ )	$x_i M_i$
4000	2500	0.191	477.5
8000	7500	0.381	2857.5
7000	12,500	0.333	4126.5
2000	17,500	0.095	1662.5
$\Sigma = 21,000$		$\Sigma = 1$	$\Sigma = 9160$

$$M_n = \Sigma x_i M_i = 9160 \text{ g/mol}$$

# Answer

- (2) Calculated the weight average molecular weight

Number of Chain	Average weight ( $M_i$ )	Total weight	$w_i$	$w_i M_i$
4000	2500	$10 \times 10^6$	0.0519	129.75
8000	7500	$60 \times 10^6$	0.3118	2338.50
7000	12,500	$87.50 \times 10^6$	0.4545	5681.25
2000	17,500	$35 \times 10^6$	0.1818	3181.50
$\Sigma = 21,000$		$\Sigma = 192.5 \times 10^6$	$\Sigma = 1$	$\Sigma = 11,331$

$$M_w = \Sigma w_i M_i = 11,331 \text{ g/mol}$$
$$M_w > M_n$$

# Answer

- (3) calculate the degree of polymerization



	C	H	Cl
Atomic weight (g/mol)	12.01	1.01	35.45

$$\bar{m} = 2(12.01) + 3(1.01) + 35.45$$

$$\bar{m} = 62.50 \text{ g/mol}$$

$$n_n = \frac{\overline{M}_n}{\bar{m}} = 146.56 \rightarrow 146$$

## $M_z$ : Z-Average Mol. Wgt.

- The z-average molecular weight (molar mass) is

$$M_z = \frac{\sum N_i M_i^3}{\sum N_i M_i^2}$$

- $M_z$  is especially **sensitive** to the presence of high-MW chains.
- $M_z$  may be determined directly by **sedimentation equilibrium** (ultracentrifugation) and **light scattering**.

## $M_v$ : Viscosity-Average Mol. Wgt.

- The viscosity-average molecular weight (molar mass) is :

$$M_v = \left[ \frac{\sum N_i M_i^{(1+a)}}{\sum N_i M_i} \right]^{1/a}$$

where the exponent  $a$  ( $0.5 \leq a \leq 2.0$ ) is determined by the polymer, solvent, and temperature.

- For typical polymers,  $M_w > M_v > M_n$ .
- $M_w = M_v$  when  $a = 1$ .
- $M_v$  may be determined indirectly by **dilute solution viscometry**.

# Polymer Size and Shape

- Most polymers are **polydisperse** – they contain more than one chain length.
- The average distribution of chain masses can be described in more than one way:
  - $M_n$ , the **number-average** molecular weight
  - $M_w$ , the **weight-average** molecular weight
  - $M_z$ , the **z-average** molecular weight
  - $M_v$ , the **viscosity-average** molecular weight
- $M_z \geq M_w \geq M_v > M_n$
- Each value is determined **certain properties** of polymer structure.

# Impact of Molecular weight on physical properties

- $M_n$ : Brittleness
- $M_w$ : Hardness
- $M_z$ : Stiffness
- $M_v$ : Molding properties

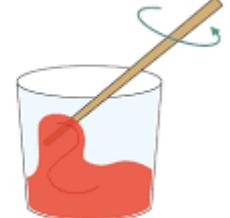


Low viscosity



Water

High viscosity



Syrup

